Why do millisecond pulsars have weaker magnetic fields compared to ordinary pulsars?

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Millisecond pulsars, with magnetic fields weaker by three to four orders compared to those of ordinary pulsars, are presumed to be neutron stars spun up by binary accretion. We expect the magnetic field to get screened by the accreted material. Our simulation of this screening mechanism shows, for the first time, that the field decreases by a purely geometric factor $\sin^{-7/2}\theta_{\rm P,i}$ before freezing to an asymptotic value, where $\theta_{\rm P,i}$ is the initial angular width of the polar cap. If $\theta_{\rm P,i}$ lies in the range 5^o–10^o, then the magnetic field diminution factor turns out to be $\sim 10^3$–10^4 in conformity with observational data.

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Pulsars—which are rotating neutron stars—are seats of strongest magnetic fields known to mankind. Magnetic fields of most pulsars are around 10^{11}–10^{13} G, whereas the typical rotation periods are about 1 s. However, there exist a handful of known pulsars with considerably smaller rotation periods, which also have much weaker magnetic fields around 10^8 G. These so-called millisecond pulsars are often found in binary systems (*1*).

It is believed that most of the neutron stars (barring possible *magnetars* (*2*)) are born with characteristics typical of ordinary pulsars. If the neutron star happens to be in a binary system,

then it is possible for it to accrete matter with angular momentum from the binary companion. Millisecond pulsars are thought to be neutron stars which have been spun up in such a binary accretion process (*3*). Since their magnetic fields are weaker by a factor 10^3–10^4 compared to the magnetic fields of ordinary pulsars, presumably the magnetic field of the neutron star also decreases during the accretion phase. Several alternative scenarios have been proposed to explain how this decrease of magnetic field takes place, starting from models relating the field evolution to the spin evolution to models based on accretion-induced Ohmic decay in the heated crustal layers (*4, 5, 6, 7, 8, 9, 10*). One possibility is that the magnetic field gets buried under the accreted matter. Although this idea has been around for a while (*11, 12, 13, 14, 15, 16, 17*), we have, for the first time, carried out a detailed 2-D simulation to test this idea. Details of our calculations are presented elsewhere (*18, 19*). One of the attractive features of this scenario is that the factor 10^3–10^4 by which the magnetic field decreases can be explained in a very elegant and simple way as arising purely out of geometric considerations.

The strong magnetic field of an accreting neutron star channelizes the accreting material to flow through the polar regions. As the magnetic field of the neutron star decreases because of the screening due to accreting material, it is less able to channelize the accretion flow and thereby the polar cap widens. One can easily find out how the angular width $\theta_{\rm P}$ of the polar cap depends on the surface magnetic field $B_{\rm s}$ of the neutron star (see, for example, Shapiro and Teukolsky (*20*)). The field line starting from $\theta_{\rm P}$ at the surface of the neutron star, with a radius $r_{\rm s}$, is the last closed field line of the dipolar field and passes through the Alfvén radius $r_{\rm A}$. It easily follows that

$$\sin\theta_{\rm P} = \left(\frac{r_{\rm s}}{r_{\rm A}}\right)^{1/2}. \qquad (1)$$

Assuming that the ram pressure of the freely in-falling accreting material at the Alfvén radius equals the magnetic pressure, a few steps of easy algebra give

$$r_{\rm A} = (2GM)^{-1/7} r_{\rm s}^{12/7} B_{\rm s}^{4/7} \dot{M}^{-2/7}, \qquad (2)$$

where M is the mass of the neutron star and $\dot{M}$ the accretion rate. It follows from (1) and (2) that

$$\sin\theta_{\rm P} \propto B_{\rm s}^{-2/7}. \tag{3}$$

This is how the polar cap widens with the weakening magnetic field until $\theta_{\rm P}$ becomes equal to 90^o when (3) obviously ceases to hold. On taking $M = 10^{33}$ gm, $\dot{M} = 10^{-8} M_\odot$ yr^{-1}, $r_{\rm s} = 10$ km, $B_{\rm s} = 10^{12}$ G, we find from (2) that $r_{\rm A} \approx 300$ km. Substituting this in (1), we conclude that the initial polar cap angle is of order 10^o.

The accreting materials falling through the two polar caps flow horizontally towards the equator in both the hemispheres. At the equator, the opposing materials flowing in from the two poles meet, sink underneath the surface (inducing a counter-flow underneath the equator-ward flow at the surface) and eventually settle radially on the neutron star core. With a suitably specified flow having these characteristics, we have studied kinematically how the magnetic field evolves with time, taking into account the fact that the polar cap width changes with the evolution of the magnetic field, thereby altering the velocity field also. Fig. 1 shows the velocity field at an early stage (A) and at a late stage (B). We find that the equator-ward flow near the surface seen in Fig. 1A is quite efficient in burying the magnetic field underneath the surface. However, when the polar cap opens to 90^o, the accretion becomes spherical and radial, as seen in Fig. 1B. It is found that such accretion is not efficient in burying the magnetic field any further. Fig. 2 shows magnetic field lines at different stages of evolution, whereas Fig. 3 plots the surface magnetic field at 45^0 as a function of time. Clearly, the magnetic field at the surface of the neutron star keeps decreasing until the polar cap opens to 90^o, after which the magnetic field is essentially frozen, since the radial accretion cannot screen it any further. If $\theta_{\rm P,i}$ is the initial polar cap width, then it follows from (3) that the magnetic field would decrease by a factor $(\sin 90^o/\sin\theta_{\rm P,i})^{7/2}$ from its initial value before it is frozen to an asymptotic value. On taking $\theta_{\rm P,i}$ in the range 5^o–10^o, this factor turns out to be about 10^3–10^4, exactly the factor by

which the magnetic fields of millisecond pulsars are weaker compared to the magnetic fields of ordinary pulsars.

Put another way, the magnetic field freezes when the Alfvén radius becomes equal to the neutron star radius. The asymptotic value of the surface magnetic field can be found directly from (2) by setting $r_{\rm A}$ equal to $r_{\rm s}$, which gives

$$B_{\rm asymp} = (2GM)^{1/4} \dot{M}^{1/2} r_{\rm s}^{-5/4}. \tag{4}$$

On using the various standard values mentioned before, we find $B_{\rm asymp} \approx 10^8$ G. When the magnetic field falls to this value, it can no longer channelize the accretion flow, resulting in the flow becoming isotropic. Such a flow is unable to screen the magnetic field any further. After the accretion phase is over, the neutron star appears as a millisecond pulsar with this magnetic field. We propose this as the reason why millisecond pulsars are found with magnetic fields of order 10^8 G.

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Fig. 1. Velocity fields induced inside the neutron star due to accreting material settling on the surface. The arrows indicate the flow velocities, whereas the dashed lines are contours of constant $\nabla.(\rho\mathbf{v})$. The velocity field is specified in the code in such a way that it keeps changing with the evolution of the magnetic field. The details of how we do this are provided elsewhere (*19*). Here we show the velocity fields at two different instants: (A) when the magnetic field is strong and the polar cap angle is small (at a relatively early stage); and (B) when the magnetic field has become much weaker and the polar cap has opened up (at a relatively late stage). The dashed lines (contours of constant $\nabla.(\rho\mathbf{v})$) indicate regions which are sources of new material due to accretion. We see in (A) that the new material is dumped in a narrow polar cap, inducing an equator-ward flow just below the surface. After reaching the equator where this flow meets the oppositely-directed flow from the other pole, the flow sinks underneath the surface, induces a counter-flow and eventually settles on the core of the neutron star. On the other hand, we see in (B) that the new material falls isotropically all over the surface and the induced flow is radially inward. The horizontal flow shown in (A) is expected in reality to be confined only within 1% of the neutron star radius immediately below its surface (*18*). Here we have inflated that layer to 10% of radius for easy visualization.

Fig. 2. Magnetic field lines during different phases of evolution. The dashed lines indicate the contours of constant $\nabla.(\rho\mathbf{v})$ at the same instants. The topmost panel shows the initial magnetic field, which is evolved by solving the induction equation numerically, with the specified velocity field which keeps changing as the magnetic field weakens. (The velocity fields at an early stage and at a late stage are shown in Fig. 1.) The details of the numerical code are given in the Appendix of our earlier paper (*18*), which also discusses the various boundary conditions used. The code has been adopted from a code which was used extensively for studying the evolution of the solar magnetic fields (*21, 22*).

Fig. 3. We show how the magnetic field at the surface at 45^0 latitude changes with time (solid line). The evolution of the polar cap angle θ_P with time is also indicated (dashed line). It is clear the magnetic field decreases rapidly till the polar cap opens to about 90^0, after which the decay of the magnetic field is significantly halted.

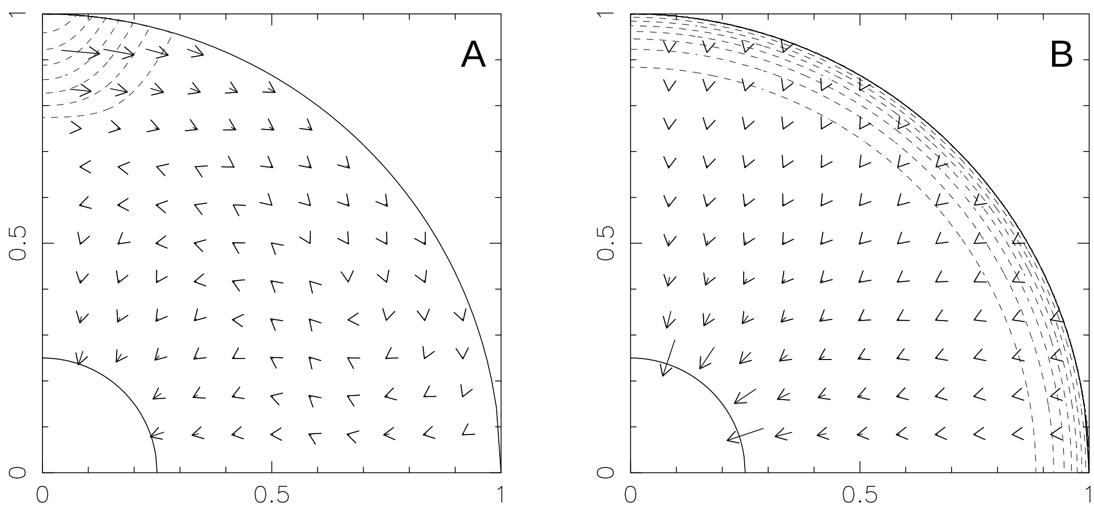

Figure 1:

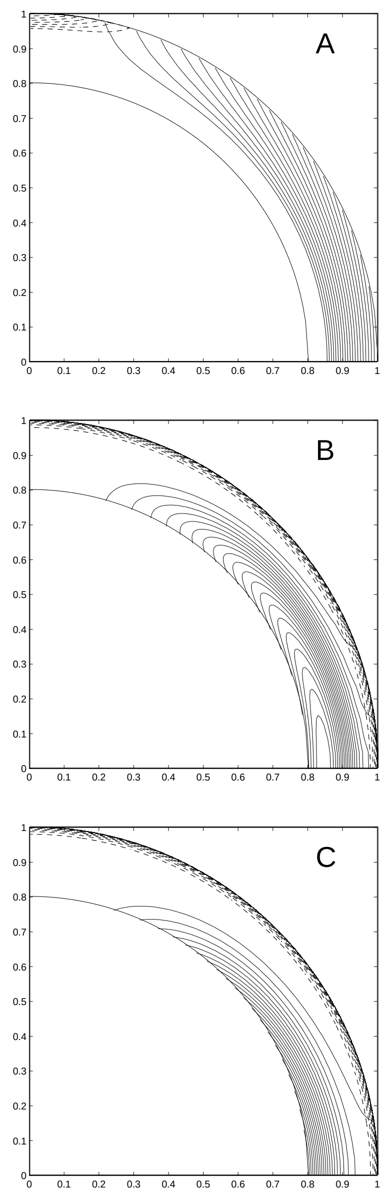

Figure 2:

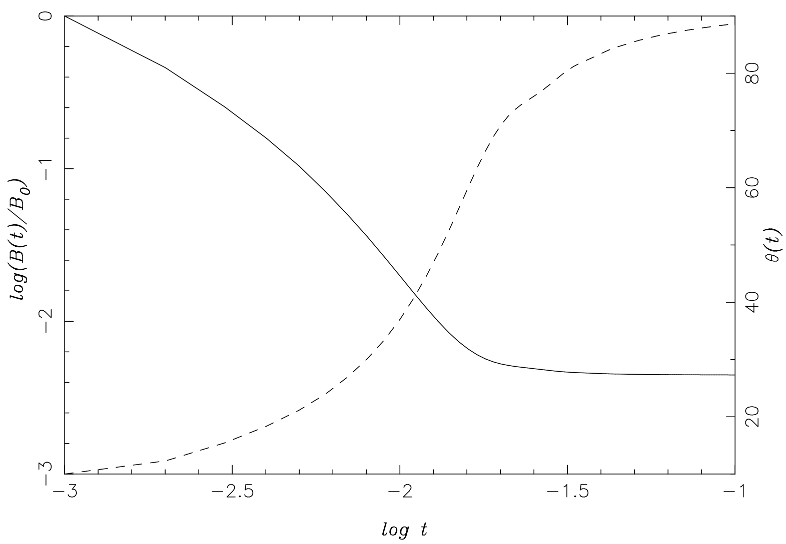

Figure 3: